Sheet 1 — Solutions

MA180 / MA185 / MA190 Mathematics

1. An x-intercept occurs when $f(x) = 0$. Since a rational function is zero when its numerator is zero (and its denominator is nonzero), we solve

$$x^2 - 7x - 8 = 0.$$

Factoring gives

$$x^2 - 7x - 8 = (x-8)(x+1),$$

so

$$x = 8 \qquad \text{or} \qquad x = -1.$$

Neither value makes the denominator zero. Therefore the x-intercepts are

$$\boxed{(8,0) \text{ and } (-1,0)}.$$

Thus the correct choice is $\boxed{\text{(a)}}$.

2. A y-intercept would require the function to be defined at $x = 0$. However,

$$f(0) = \frac{-8}{0},$$

which is undefined. Hence the graph has no y-intercept.

Therefore the correct choice is

$$\boxed{\text{(c) No.}}$$

3. For the right-hand limit as $x \to 1^+$, the relevant formula is $f(x) = 2 - x^2$. Thus

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (2 - x^2) = 2 - 1^2 = 1.$$

Also,

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x = 1,$$

so in fact

$$\lim_{x \to 1} f(x) = 1,$$

even though $f(1) = 3$.

Therefore the correct choice is

$$\boxed{\text{(a)}}$$

since

$$\boxed{\lim_{x \to 1^+} f(x) = 1}.$$

4. When x approaches -8 from the left, x lies in the interval

$$[-9, -8).$$

For every x in this interval,

$$\lfloor x \rfloor = -9.$$

Therefore

$$\boxed{\lim_{x \to -8^-} \lfloor x \rfloor = -9}.$$

Thus the correct choice is $\boxed{\text{(b)}}$.

5. Each formula is a polynomial, so the only possible point of discontinuity is at $x = 2$. From the left, including the value at $x = 2$,

$$f(2) = 2^2 + 2r(2) = 4 + 4r.$$

From the right,

$$\lim_{x \to 2^+} f(x) = s(2^2) + 4 = 4s + 4.$$

Continuity requires

$$4 + 4r = 4s + 4.$$

Subtracting 4 and dividing by 4 gives

$$r = s.$$

Hence the function is continuous if and only if

$$\boxed{r = s}.$$

Thus the correct choice is $\boxed{\text{(a)}}$.

6. Apply the Euclidean algorithm:

$$\begin{aligned} 1989 &= 4(476) + 85, \\ 476 &= 5(85) + 51, \\ 85 &= 1(51) + 34, \\ 51 &= 1(34) + 17, \\ 34 &= 2(17) + 0. \end{aligned}$$

The last nonzero remainder is 17. Therefore

$$\boxed{\gcd(1989, 476) = 17}.$$

7. Applying the Euclidean algorithm gives

$$\begin{aligned}
334 &= 6(51) + 28, & r_1 &= 28, \\
51 &= 1(28) + 23, & r_2 &= 23, \\
28 &= 1(23) + 5, & r_3 &= 5, \\
23 &= 4(5) + 3, & r_4 &= 3, \\
5 &= 1(3) + 2, \\
3 &= 1(2) + 1, \\
2 &= 2(1) + 0.
\end{aligned}$$

Thus

$$\gcd(334, 51) = 1$$

and, in particular,

$$\boxed{r_4 = 3}.$$

8. First apply the Euclidean algorithm:

$$\begin{aligned}
41 &= 2(18) + 5, \\
18 &= 3(5) + 3, \\
5 &= 1(3) + 2, \\
3 &= 1(2) + 1, \\
2 &= 2(1) + 0.
\end{aligned}$$

The last nonzero remainder is 1, so

$$\gcd(41, 18) = 1.$$

Hence 41 and 18 are coprime.

Now back substitute:

$$\begin{aligned}
1 &= 3 - 2 \\
&= 3 - (5 - 3) \\
&= 2(3) - 5 \\
&= 2(18 - 3 \cdot 5) - 5 \\
&= 2(18) - 7(5) \\
&= 2(18) - 7(41 - 2 \cdot 18) \\
&= -7(41) + 16(18).
\end{aligned}$$

Comparing this with

$$41x + 18y = 1,$$

we obtain

$$x = -7, \qquad y = 16.$$

Therefore

$$\boxed{y = 16}.$$