Sheet 1

MA180 / MA185 / MA190 Mathematics

1. What are the x-intercepts of the function

$$f(x) = \frac{x^2 - 7x - 8}{x^4}\,?$$

   (a) $(8, 0)$, $(-1, 0)$
   (b) $(-8, 0)$, $(1, 0)$
   (c) $(7, 0)$, $(1, 0)$
   (d) $(-8, 0)$, $(-1, 0)$
   (e) None of the above.

2. Does the function

$$f(x) = \frac{x^2 - 7x - 8}{x^2}$$

   have a y-intercept?

   (a) Yes, it has a y-intercept at $(0, 1)$.
   (b) Yes, it has a y-intercept at $(0, -8)$.
   (c) No.
   (d) None of the above.

3. Let

$$f(x) = \begin{cases} x & \text{if } x < 1, \\ 3 & \text{if } x = 1, \\ 2 - x^2 & \text{if } 1 < x \leq 2, \\ x - 3 & \text{if } x > 2. \end{cases}$$

   Then $f(x)$ satisfies:

   (a) $\lim\limits_{x \to 1^+} f(x) = 1$.
   (b) $\lim\limits_{x \to 1^+} f(x) = 3$.
   (c) $\lim\limits_{x \to 1} f(x) = 3$.
   (d) $\lim\limits_{x \to 1} f(x) = 2$.
   (e) None of the above.

4. The greatest integer function is defined by

$$\lfloor x \rfloor = \text{the largest integer less than or equal to } x.$$

For instance,

$$\lfloor 2.4 \rfloor = 2, \qquad \lfloor 3 \rfloor = 3, \qquad \lfloor -0.5 \rfloor = -1.$$

In particular, if n is any integer, then

$$\lfloor x \rfloor = n \qquad \text{for all } x \in [n, n+1).$$

Evaluate

$$\lim_{x \to -8^-} \lfloor x \rfloor,$$

if it exists.

(a) The limit does not exist.

(b) -9.

(c) -8.

(d) -8.5.

5. The function

$$f(x) = \begin{cases} x^2 + 2rx, & x \leq 2, \\ sx^2 + 4, & x > 2 \end{cases}$$

is continuous if and only if

(a) $r = s$.

(b) $r = s - 5$.

(c) $s = 2r$.

(d) $s = 4r + 1$.

(e) None of the above.

6. Use the Euclidean algorithm to determine the greatest common divisor of 1989 and 476.

7. Use the Euclidean algorithm to compute the greatest common divisor of 334 and 51.

   Let r_i be the remainder found during the ith step, so that, for instance, $r_1 = 28$.

   What is r_4?

8. Use the Euclidean algorithm to verify that 41 and 18 are coprime.

   Then use back substitution to determine integers x and y such that

$$41x + 18y = 1.$$

   What is y?