Solutions

1. a. True.

   b. False.

   c. False.

   d. True.

   e. True.

2. a. No.

   b. Yes, $\begin{pmatrix} 1 & 3 \\ 6 & 9 \end{pmatrix}$.

   c. No.

   d. Yes, $\begin{pmatrix} 5 & -2 & 7 \\ 11 & -6 & 15 \end{pmatrix}$.

3. a.

$$\det(A) = 5 - 3 = 2 \neq 0.$$

$$A^{-1} = \begin{pmatrix} \frac{1}{2} & -\frac{3}{2} \\ -\frac{1}{2} & \frac{5}{2} \end{pmatrix}.$$

   b.

$$BC = \begin{pmatrix} 1 & x & 0 \\ 2+y & 1 & 0 \\ 0 & x & 1 \end{pmatrix}.$$

   We need $x = 0$ and $y = -2$.

4. a.

$$T\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 2-2+3 \\ 4+1 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}.$$

   b.

$$T(e_1) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \qquad T(e_2) = \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \qquad T(e_3) = \begin{pmatrix} -3 \\ 0 \end{pmatrix}.$$

   If $A = \begin{pmatrix} 1 & -2 & -3 \\ 2 & 1 & 0 \end{pmatrix}$, then $T = T_A$.

   c. Need

$$A\begin{pmatrix} x \\ y \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}.$$

$$A\begin{pmatrix} x \\ y \\ 0 \end{pmatrix} = \begin{pmatrix} x-2y \\ 2x+y \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}.$$

   This is another linear system:

$$B = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}, \qquad v = \begin{pmatrix} x \\ y \end{pmatrix}, \qquad b = \begin{pmatrix} -1 \\ 3 \end{pmatrix}.$$

$$\left(\begin{array}{cc|c} 1 & -2 & -1 \\ 2 & 1 & 3 \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} 1 & -2 & -1 \\ 0 & 5 & 5 \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} 1 & -2 & -1 \\ 0 & 1 & 1 \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1 \end{array}\right)$$

$$x = y = 1.$$

5. a.

$$\begin{aligned} 2 + 2 + t = 3 \quad &\Rightarrow \quad t = -1. \\ 1 - 2 - 1 = 0 \quad &\times \end{aligned}$$

False, $(1, 2, t)$ does not satisfy the first two equations.

b.

$$2t + 3 + 3 = 3 \quad \Rightarrow \quad t = -\tfrac{3}{2}.$$

False, $(t, 3, 3)$ does not satisfy the first two equations.

c. (True by instructions.)

$$\begin{aligned} 2 + t + 3 = 3 \quad &\Rightarrow \quad t = -2. \\ 1 + 2 - 3 = 0 \quad &\checkmark \\ 3 - 4 + 3 = 2 \quad &\checkmark \end{aligned}$$

True.