MA180/MA185/MA190 Sample algebra test November 2025

Name: _____

Student ID number: _____

Module code: ◯ MA180 ◯ MA185 ◯ MA190

(Optional:) LENS report: ◯ Yes ◯ No

Instructions. Provide your answers in the spaces provided. If you run out of space, extra paper will be provided. Be sure to include your name and student ID number on all additional sheets of paper that you submit.

If you have a LENS report that stipulates extra time, then please indicate this above and answer any four out of the five questions below.

All other students, please answer all five questions.

1. ($10 = 2 + 2 + 2 + 2 + 2$ marks) (a) Suppose the function $f : \mathbb{Z}_{26} \to \mathbb{Z}_{26}$ defined by $f(x) = x + 15$ is used for encoding one-unit messages over a 26-letter alphabet. Then the decoding function is $g(x) = 15x + 11$. ◯ True ◯ False

   (b) Suppose the function $f : \mathbb{Z}_{26} \to \mathbb{Z}_{26}$ defined by $f(x) = x+20$ is used for encoding one-unit messages over a 26-letter alphabet. Then the decoding function is $g(x) = x + 6$.
   ◯ True ◯ False

   (c) The following congruence holds: $7^{41} \equiv 7 \pmod{50}$
   ◯ True ◯ False

   (d) The following congruence holds: $9^{60} \equiv 1 \pmod{50}$ ◯ True ◯ False

   (e) The following linear system admits a solution:

$$\begin{cases} 2x + 3y &= 3 \\ 4x + 6y &= 0 \end{cases}$$

   ◯ True ◯ False

2. (10 = 2 + 2 + 3 + 3 marks) For each of the following operations, determine whether it can be carried out or not. If it can be computed, do so. If it cannot, explain why.

(a) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & 4 \end{pmatrix}$ ◯ Yes ◯ No

(b) $\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$ ◯ Yes ◯ No

(c) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & 4 \end{pmatrix}$ ◯ Yes ◯ No

(d) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ 0 & 3 \end{pmatrix}$ ◯ Yes ◯ No

3. (10 marks) (a) Show that the following matrix is invertible and determine its inverse.

$$A = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$$

(b) Let $B = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} 0 & 1 & x \\ 1 & -1 & y \\ -1 & 1 & 0 \end{pmatrix}$. Determine x and y such that $BC = I_{3\times 3}$, that is such that $C = B^{-1}$.

4. (10 marks) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation defined as follows:

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2x + 3y - z \\ x + z \end{pmatrix}.$$

(a) Determine the image under T of the point $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$.

(b) Determine the matrix of the transformation T with respect to the standard basis of $\mathbb{R}^3$.

(c) Determine x and z such that $T\begin{pmatrix} x \\ 0 \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

5. (10 marks) Consider the following system of linear equations

$$(*) \quad \begin{cases} x + y + z & = 6 \\ x + 3y - 5z & = 6 \\ 5x - 3y - z & = 0 \end{cases}.$$

For each of (a),(b),(c) decide whether there exists t that completes the given vector to a solution of $(*)$ and compute t, or explain why it does not exist. Exactly one of (a),(b) and (c) can be completed to a solution.

(a) $(2, 2, t)$ can be completed to a solution of $(*)$ ◯ True ◯ False

(b) $(t, 3, 2)$ can be completed to a solution of $(*)$ ◯ True ◯ False

(c) $(2, t, 1)$ can be completed to a solution of $(*)$ ◯ True ◯ False

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