

Name: _____

Student ID number: _____

Module code: ☐ MA180 ☐ MA185 ☐ MA190(Optional:) LENS report: ☐ Yes ☐ No

Instructions. Provide your answers in the spaces provided. If you run out of space, extra paper will be provided. Be sure to include your name and student ID number on all additional sheets of paper that you submit.

If you have a LENS report that stipulates extra time, then please indicate this above and answer any four out of the five questions below.

All other students, please answer all five questions.

1. (10 = 2 + 2 + 2 + 2 + 2 marks) (a) The following holds: $4|20$ ☐ True ☐ False

(b) The greatest common divisor of 6 and 20 is 2. ☐ True ☐ False

(c) The greatest common divisor of 12 and 30 is 6. ☐ True ☐ False

(d) There exist integers x, y such that $4x + 10y = 1$. ☐ True ☐ False

(e) There exist integers x, y such that $2x + 5y = 1$. ☐ True ☐ False

2. (10 = 2 + 2 + 3 + 3 marks) For each of the following congruences, determine x as an integer in $\{0, 1, \dots, m-1\}$ where m is the relevant modulus.

(a) $x \equiv 4 + 7 \pmod{8}$

(b) $x \equiv 3 \cdot 7 \pmod{8}$

(c) $9x \equiv 8 \pmod{7}$

(d) $3x \equiv 2 \pmod{5}$

3. (5 + 5 marks) (a) Use Euclid's algorithm to find $\gcd(19, 5)$.

- (b) Hence or otherwise, find $5^{-1} \pmod{19}$ as an integer in $\{0, 1, \dots, 18\}$. Justify your answer.

4. (10 = 4 + 2 + 4 marks) (a) Use Fermat's Little Theorem to find $x \in \{0, 1, \dots, 6\}$ such that $x \equiv 4^{13} \pmod{7}$.

- (b) Decompose 140 as product of prime powers.

- (c) Hence or otherwise, compute $\phi(140)$, that is the number of integers between 0 and 139 that are coprime with 140.

5. ($10 = 2 + 2 + 2 + 4$ marks) Consider the following system of congruences

$$(*) \quad \begin{cases} x \equiv 2 \pmod{3} \\ x \equiv 3 \pmod{4} \\ x \equiv 2 \pmod{5} \end{cases} .$$

For each of (a),(b),(c) decide whether x is a solution to the system or explain why it is not. (Exactly one of the three is a solution.)

- (a) $x = 11$ is a solution to $(*)$ ☐ True ☐ False

- (b) $x = 32$ is a solution to $(*)$ ☐ True ☐ False

- (c) $x = 47$ is a solution to $(*)$ ☐ True ☐ False

- (d) Use the information above to find an integer between 100 and 150 that is a solution to the system. Briefly justify your answer.

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