Eigenvalues and vectors of inverses!

Ex: $A = \begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix}$ $\lambda = 5$ $v = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$\lambda = -2$ $v = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\det(A) = -10$

$$A^{-1} = \frac{-1}{10}\begin{pmatrix} 2 & -3 \\ -4 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} -2 & +3 \\ +4 & -1 \end{pmatrix}.$$

We could find the eigenvectors and values for B (won't be any diff. from A^{-1}) The defining equation for A^{-1} is

$$A \cdot A^{-1} = I.$$

For inverses, the eigenvectors stay the same. The eigenvalues get inverted.

$$\frac{-1}{10}\begin{pmatrix}2 & -3\\ -4 & 1\end{pmatrix}\begin{pmatrix}1\\ -1\end{pmatrix} = \frac{-1}{10}\cdot\begin{pmatrix}5\\ -5\end{pmatrix} = \frac{-1}{2}\begin{pmatrix}1\\ -1\end{pmatrix}$$

$$\frac{-1}{10}\begin{pmatrix}2 & -3\\ -4 & 1\end{pmatrix}\begin{pmatrix}3\\ 4\end{pmatrix} = \frac{-1}{10}\begin{pmatrix}-6\\ -8\end{pmatrix} = \frac{1}{5}\begin{pmatrix}3\\ 4\end{pmatrix}.$$

A matrix A is invertible if and only if 0 is <u>not</u> an eigenvalue of A.

① Find a value for $x \in \{0, 1, \ldots, 293\}$ such that

$$x \equiv 5^{8487} \pmod{294}$$

Not prime

$294 = 2 \cdot 147 = 2 \cdot 3 \cdot 49 = 2 \cdot 3 \cdot 7^2$

Euler's Theorem: If $\gcd(a, m) = 1$, then

$$a^{\varphi(m)} \equiv 1 \pmod{m}.$$

$\varphi(294) = (2-1) \cdot (3-1) \cdot 7 \cdot (7-1)$

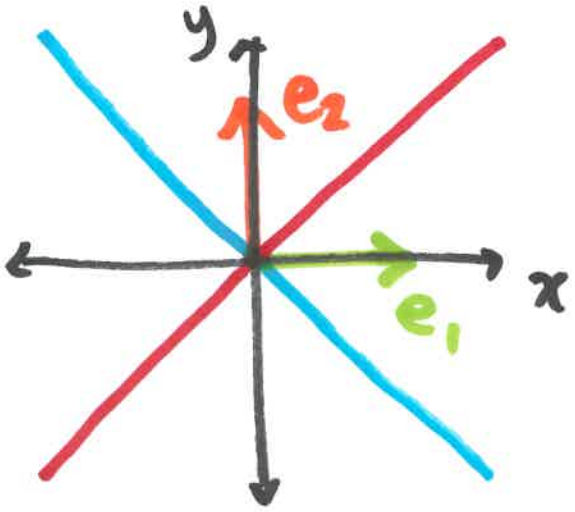
$= 2 \cdot 7 \cdot 6 = 84.$

$$8487 = 3 + 8484$$
$$= 3 + 84 \cdot (101)$$

$$5^{8487} = 5^3 \cdot (5^{84})^{101} \equiv 5^3 \cdot 1^{101} = \boxed{125} \pmod{294}$$

$$x = 125.$$

(2) Let $f: \mathbb{R}^2 \to \mathbb{R}^2$ be the reflection about $y = x$ and $g: \mathbb{R}^2 \to \mathbb{R}^2$ the reflection about $y = -x$. Determine matrices for the compositions $f \circ g$ and $g \circ f$.



$$F = \begin{pmatrix} f(e_1) & f(e_2) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$G = \begin{pmatrix} g(e_1) & g(e_2) \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

$$f(v) = F \cdot v \qquad g(v) = G \cdot v$$

$$(f \circ g)(v) = f(g(v)) = f(G \cdot v) = F \cdot G \cdot v$$

composition — product

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

F G — match!

$$(g \circ f)(v) = G \cdot F \cdot v$$

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}.$$

(3) Find eigenvectors and values of

$$A = \begin{pmatrix} 5 & 1 \\ -2 & 2 \end{pmatrix}.$$

Is A invertible? If so what are the eigenvectors and values?