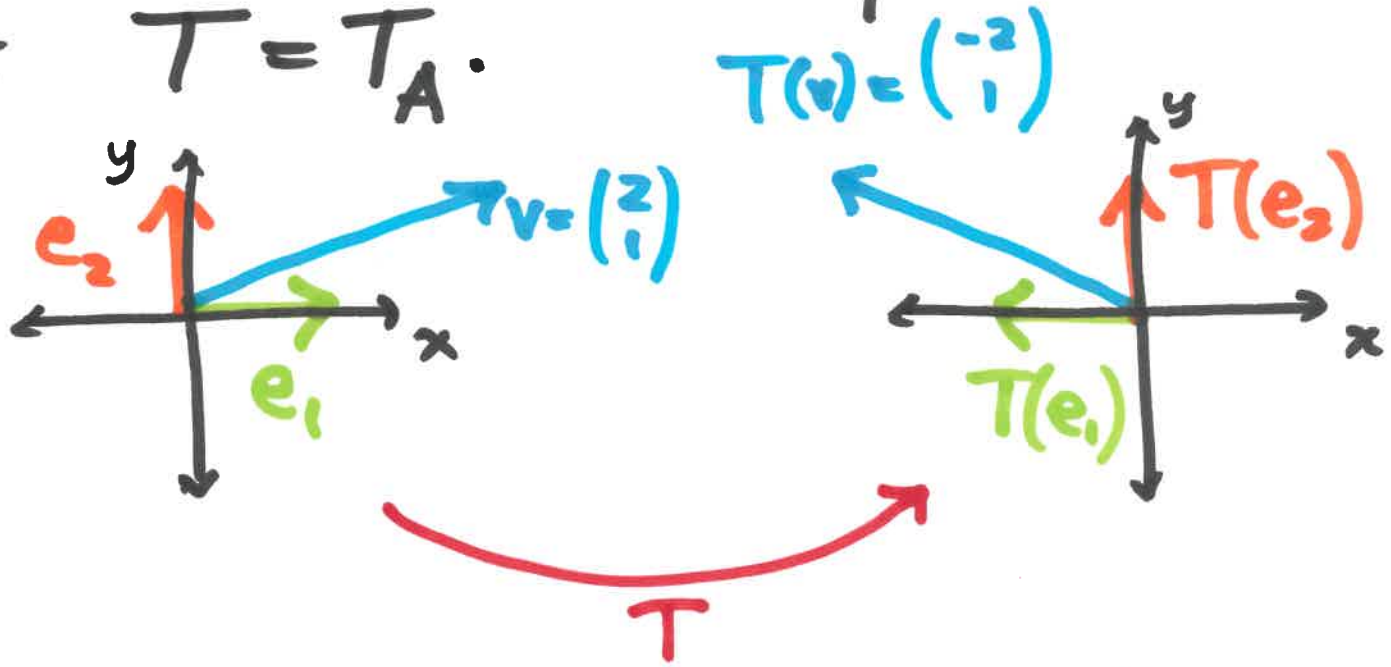
Ex: ~~Find~~ $T: \mathbb{R}^2 \to \mathbb{R}^2$ is given by a reflection about the y-axis. Find A such that $T = T_A$.



$$A = \begin{pmatrix} T(e_1) & T(e_2) \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$T(v) = Av = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

Ex: $T: \mathbb{R}^2 \to \mathbb{R}^2$ is the reflection about the line $y=x$. Find A such that $T = T_A$.

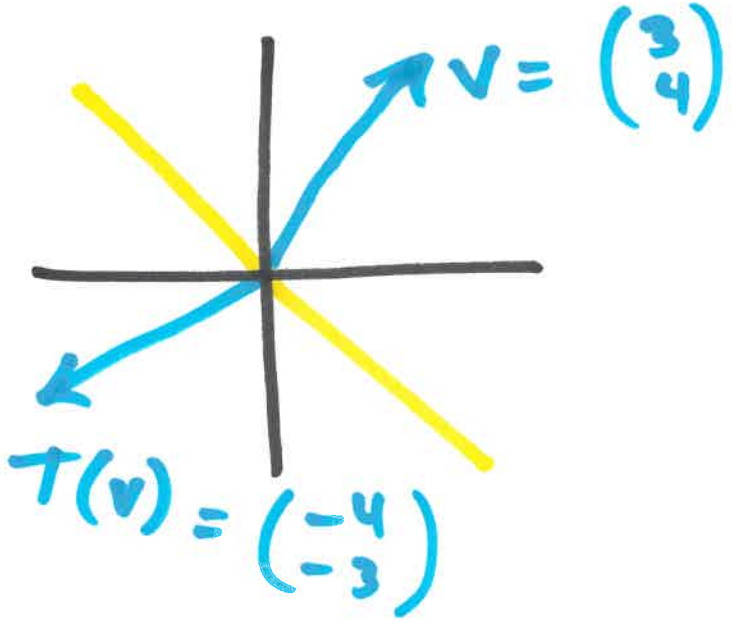


$$A = \begin{pmatrix} T(e_1) & T(e_2) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Ex: $T: \mathbb{R}^2 \to \mathbb{R}^2$ is the reflection about the line $y = -x$. What is the image of $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ under T?

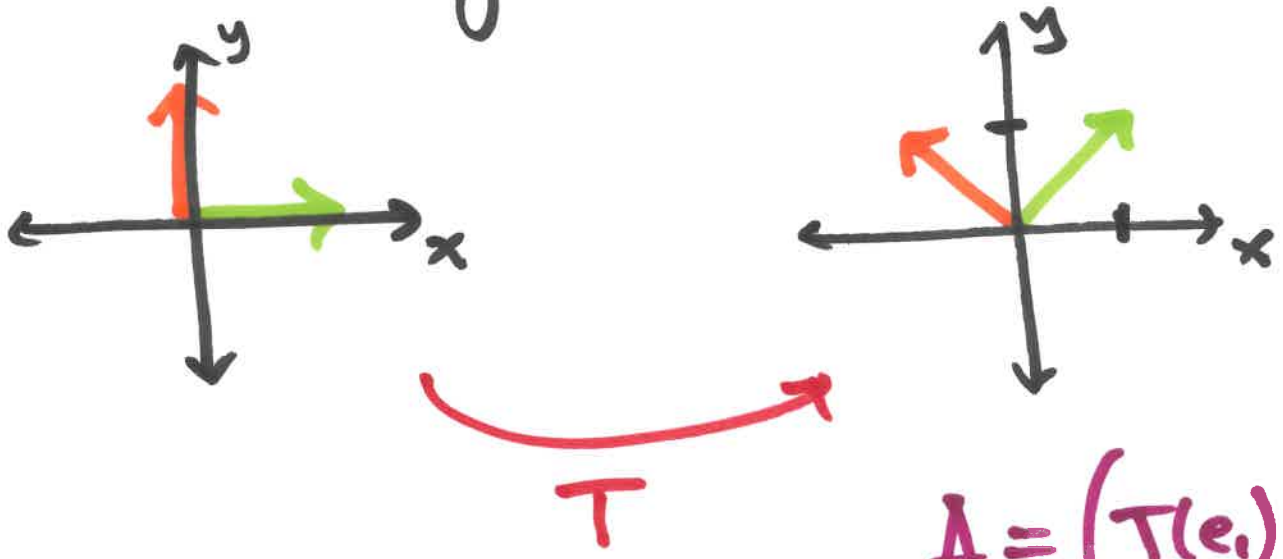
$T\begin{pmatrix} 3 \\ 4 \end{pmatrix} = ?$

$A = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$.



Rotations are also linear transforms.

Ex: $T: \mathbb{R}^2 \to \mathbb{R}^2$ is the counterclockwise rotation by 45°. Find A such that $T = T_A$.





$a^2 + 2a^2 = 1$

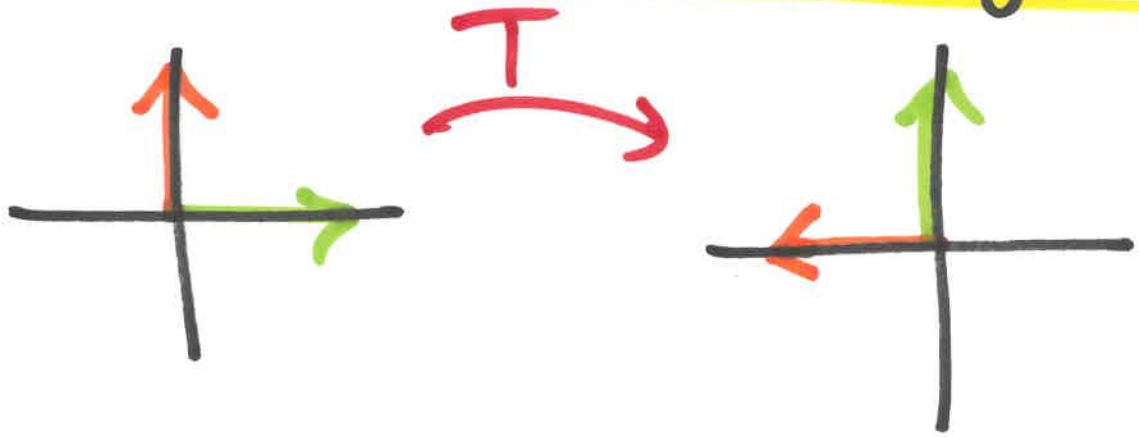
$$A = \begin{pmatrix} T(e_1) & T(e_2) \end{pmatrix} = \begin{pmatrix} \sqrt{2}^{-1} & -\sqrt{2}^{-1} \\ \sqrt{2}^{-1} & \sqrt{2}^{-1} \end{pmatrix}$$

We can scale A and get

$$A' = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

Ex: Rotate instead by 90° counterclockwise

Can think of composing T twice.



$$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

In general, we can rotate by θ radians using the "standard" rotation matrix:

(in counterclockwise)

$$A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

Algebraically what is composing of linear transformations?

$$T_A : \mathbb{R}^n \longrightarrow \mathbb{R}^m$$

$$T_B : \mathbb{R}^m \longrightarrow \mathbb{R}^l$$

$$\mathbb{R}^n \xrightarrow{T_A} \mathbb{R}^m \xrightarrow{T_B} \mathbb{R}^l$$

$$v \longmapsto Av \qquad B(Av) = \underline{\underline{B \cdot A \cdot v}}$$

$$\| \qquad\qquad \|$$

$$u \longmapsto Bu$$

Composition of linear transformations, $T_B \circ T_A$, is given by matrix multipl., $B \cdot A$.

Ex: A $T: \mathbb{R}^2 \to \mathbb{R}^2$ $\quad T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x+y \\ x-y \end{pmatrix}$

B $S: \mathbb{R}^2 \to \mathbb{R}^2$ $\quad S\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ -x \end{pmatrix}$

Compute their compositions:

$$S\left(T\begin{pmatrix} x \\ y \end{pmatrix}\right) = S\begin{pmatrix} \overbrace{2x+y}^{x'} \\ \underbrace{x-y}_{y'} \end{pmatrix} = S\begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$= \begin{pmatrix} y' \\ -x' \end{pmatrix} = \begin{pmatrix} x-y \\ -2x-y \end{pmatrix}.$$

Find matrices for T and S:

$$A = \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$S(T\begin{pmatrix} x \\ y \end{pmatrix}) = \begin{pmatrix} x-y \\ -2x-y \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & -1 \\ -2 & -1 \end{pmatrix}$$

$$B \cdot A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -2 & -1 \end{pmatrix}$$