Vectors

Can think of vectors as matrices.

For us, the default is a column vector, that is an $n \times 1$ matrix.

$$v = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$ is a col. vec. (3×1 matrix)

A row vector is a $1 \times n$ matrix.

Because vectors are matrices, we can multiply (when it makes sense) vectors & mats.

Ex:

$$Av = \begin{pmatrix} 2 & 0 & 3 \\ 1 & -1 & 2 \\ 4 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2\cdot 1 + 0\cdot 2 + 3\cdot 3 \\ 1\cdot 1 + (-1)\cdot 2 + 2\cdot 3 \\ 4\cdot 1 + 2\cdot 2 + 1\cdot 3 \end{pmatrix} = \begin{pmatrix} 11 \\ 5 \\ 11 \end{pmatrix}$$

3×3 $\quad$ 3×1



Consider the linear system:

$$\begin{cases} 2x + 3y + z = 7 \\ 2x + y + 3z = 9 \\ 4x + 2y + 5z = 16 \end{cases}$$

$$A = \begin{pmatrix} 2 & 3 & 1 \\ 2 & 1 & 3 \\ 4 & 2 & 5 \end{pmatrix} \quad b = \begin{pmatrix} 7 \\ 9 \\ 16 \end{pmatrix} \quad v = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Mod. arith.
$ax \equiv b \pmod{m}$

$Av = b$ ← Equation of matrices.

$$\begin{pmatrix} 2 & 3 & 1 \\ 2 & 1 & 3 \\ 4 & 2 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \overset{MM}{=} \begin{pmatrix} 2x + 3y + z \\ 2x + y + 3z \\ 4x + 2y + 5z \end{pmatrix} = \begin{pmatrix} 7 \\ 9 \\ 16 \end{pmatrix}$$

We want to solve $Av = b$ for v.

We saw one way: Gaussian elimination.

Another way: $v = A^{-1}b$ (when possible!)

Algebraic identities:

Assume A, B, and C are matrices of appropriate dimensions.

1. $A + B = B + A$ — commutativity
2. $(A + B) + C = A + (B + C)$ — associativity
3. $(AB)C = A(BC)$ — associativity

④ $A(B+C) = AB + AC$

$(A+B)C = AC + BC$ distributivity

Three special (kinds of) matrices.

① Zero matrix: $\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$ All zeros.

② Diagonal matrix: square matrices, # rows = # cols, where all non-zero entries are on the diagonal, these are have position (i,i) for all i.

$\begin{pmatrix} 3 & 0 \\ 0 & -1 \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\pi \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

(3) Identity matrix: diagonal matrices with all 1s on the diagonal.

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Identities: Z zero matrix, I identity mat.

$$\underbrace{A+Z=A,\quad A\cdot Z=Z'}_{Z\approx 0},\qquad \underbrace{A\cdot I=A.}_{I\approx 1.}$$

Inverses

A matrix A has an inverse, B, if

$$A\cdot B=B\cdot A=I \qquad (I = \text{identity})$$

$ax \equiv b \pmod m$

a^{-1} exists if there is a c: $ac \equiv 1 \pmod m$

~~The inverse~~ If an inverse exists, we say that A is invertible or nonsingular.
(If one does not exist, then A is singular.)
It's written A^{-1}.

L.S. $Av = b$

$A^{-1}Av = A^{-1}b$

$Iv = A^{-1}b$

$\boxed{v = A^{-1}b}$

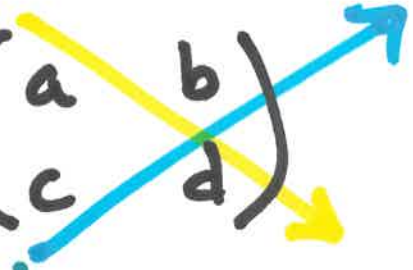
Use inverses to solve linear systems.

In this case: unique solution!

Ex: Inverse of 2×2.

Many ways to compute inverses.

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$



Define the determinant:

$$\det(A) = ad - bc$$

A has an inverse if and only if $\det(A) \neq 0$.