Number of solutions to linear systems:

Three possibilities:

1. Unique sol.
2. No sol.

Ex:

$$\left(\begin{array}{ccc|c} 1 & 3 & 1 & 7 \\ 0 & 2 & 4 & 5 \\ 0 & 0 & 0 & 1 \end{array}\right)$$

← Implies no sol.

potentially

3. Infinite* number of sol.

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Ex:

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 0 & 2 & 4 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

← Implies no unique sol. Either ∞ or 0

Matrix algebra.

A some matrix: $A = \begin{pmatrix} 1 & 2 & 3 \\ 6 & -7 & 0 \end{pmatrix}$

A has 2 rows and 3 columns. We write this: A is 2×3. I call these the dimensions sizes are also used.

Addition: Let A and B be matrices.

A+B is only defined when A and B have the same dimensions.

When dims are equal: addition is entry-wise

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 6 & -7 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 & 1 \\ \pi & 2 & -1 \end{pmatrix}$$

$$A+B = \begin{pmatrix} 1 & 2 & 4 \\ 6+\pi & -5 & -1 \end{pmatrix}$$

Scalar multiplication.

A is a matrix and $k \in \mathbb{R}$. Scalar mult. is

$k \cdot A$: apply this multiplication entry-wise.

$$(-2) \cdot A = \begin{pmatrix} -2 & -4 & -6 \\ -12 & 14 & 0 \end{pmatrix}$$

Both add. and s. mult. do not change dims.

Subtraction is $A + (-1)\cdot B$, so entry-wise!

Introduction to entries.

$$A = \begin{matrix} \\ 1 \\ 2 \end{matrix}\begin{matrix} \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{pmatrix} 0 & -1 & \pi & 4 \\ 3 & -2 & 1 & e^2 \end{pmatrix} \end{matrix}$$

We number entries by (row, column).

Entry 0 in A is in (1,1); π is in (1,3); -2 is in (2,2); -1 is in (1,2); 3 in (2,1).

These are indices of the entries.

A_{24} = the (2,4)-entry of A = the e^2.

$\boxed{A_{ij}}$ = the (i,j)-entry of A.

Multiplication. This is not entry-wise!

A is an $m \times n$ matrix, and B an $r \times s$ matrix. Mat. mul. is defined only when

$A \cdot B$: $n = r$ # of cols. of A = # of rows of B

The resulting matrix (assuming $n=r$) has dim $m \times s$

Note: this is function comp.

Ex:

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$$

$$A \cdot B = \begin{pmatrix} 1\cdot 1 + 0\cdot 3 + 2\cdot 5 & 1\cdot 2 + 0\cdot 4 + 2\cdot 6 \\ 0\cdot 1 + (-1)\cdot 3 + 3\cdot 5 & 0\cdot 2 + (-1)\cdot 4 + 3\cdot 6 \end{pmatrix} = \begin{pmatrix} 11 & 14 \\ 12 & 14 \end{pmatrix}$$

In symbols the (i,j)-entry of $A \cdot B$ is

$$(AB)_{ij} = \sum_{k=1}^{n} A_{ik} \cdot B_{kj}$$

$$= A_{i1}B_{1j} + A_{i2}B_{2j} + \cdots + A_{in}B_{nj}.$$

Ex: $A = \begin{pmatrix} 1 & 3 \\ 4 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}$

$$AB = \begin{pmatrix} -1 & 4 \\ 10 & 2 \end{pmatrix} \quad 4 = 1\cdot 1 + 3\cdot 1$$

$$BA = \begin{pmatrix} 6 & \cdot \\ \cdot & \cdot \end{pmatrix}$$

Already we see that

$$AB \neq BA.$$

Multiplication is not commutative!