

Pivot

$$\left(\begin{array}{ccc|c} \boxed{2} & 3 & 1 & 7 \\ \boxed{2} & 1 & 3 & 9 \\ \boxed{4} & 2 & 5 & 16 \end{array} \right) \xrightarrow{-R_1 \leadsto R_2}$$

$$\left(\begin{array}{ccc|c} 2 & 3 & 1 & 7 \\ \boxed{0} & -2 & 2 & 2 \\ \boxed{4} & 2 & 5 & 16 \end{array} \right)$$

Goal: Get a lot of zeros.

$$\xrightarrow{-2 \cdot R_1 \leadsto R_3}$$

$$\xrightarrow{-\frac{1}{2}R_2} \left(\begin{array}{ccc|c} 2 & 3 & 1 & 7 \\ \boxed{0} & -2 & 2 & 2 \\ 0 & -4 & 3 & 2 \end{array} \right)$$

$$\xrightarrow{4 \cdot R_2 \leadsto R_3} \left(\begin{array}{ccc|c} 2 & 3 & 1 & 7 \\ 0 & \boxed{1} & -1 & -1 \\ 0 & -4 & 3 & 2 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 2 & 3 & 1 & 7 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -1 & -2 \end{array} \right)$$

$$\xrightarrow{-R_3} \left(\begin{array}{ccc|c} \boxed{2} & 3 & 1 & 7 \\ 0 & \boxed{1} & -1 & -1 \\ 0 & 0 & \boxed{1} & 2 \end{array} \right) \text{ REF}$$

$$2x + 3y + z = 7$$

$$y - z = -1$$

$$\boxed{z = 2}$$

$$\underline{z=2} \Rightarrow \underline{y=1} \Rightarrow 2x + 3 + 2 = 7 \Rightarrow \underline{x=1}$$

back substitution (101)

This example demonstrates how we solve linear systems. We say this is via **Gaussian elimination**.

Def. A **pivot** is the first (from the left) non-zero number in a row.

The pivot is used to clear other non-zero entries, i.e. get more zeros!

When all pivots have only zeros below them, we say the matrix is in **row echelon form**. (REF) (102)

A matrix is in **reduced row echelon form (RREF)** if all entries above and below pivots are zero, and all pivots = 1.

$$\begin{pmatrix} \boxed{2} & 3 & \boxed{1} & 7 \\ 0 & \boxed{1} & -1 & -1 \\ 0 & 0 & \boxed{1} & 2 \end{pmatrix} \xrightarrow{R_3 \sim R_2} \begin{pmatrix} \boxed{2} & 3 & 1 & 7 \\ 0 & \boxed{1} & 0 & 1 \\ 0 & 0 & \boxed{1} & 2 \end{pmatrix} \xrightarrow{-R_3 \sim R_1} \begin{pmatrix} \boxed{2} & 3 & 0 & 5 \\ 0 & \boxed{1} & 0 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

$$\begin{aligned} x &= 1 \\ y &= 1 \\ z &= 2 \end{aligned}$$

$$\begin{pmatrix} 1 & 0 & 0 & | & 1 \\ 0 & 1 & 0 & | & 1 \\ 0 & 0 & 1 & | & 2 \end{pmatrix} \xleftarrow{\frac{1}{2} R_1} \begin{pmatrix} \boxed{2} & 0 & 0 & 2 \\ 0 & \boxed{1} & 0 & 1 \\ 0 & 0 & \boxed{1} & 2 \end{pmatrix} \xleftarrow{-3R_2 \sim R_1}$$

Ex:

$$\begin{cases} -2b + 3c = 1 \\ 3a + 6b - 3c = -3 \\ a + 4b + 2c = 0 \end{cases}$$

$$\begin{pmatrix} 0 & -2 & 3 & 1 \\ 3 & 6 & -3 & -3 \\ 1 & 4 & 2 & 0 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{pmatrix} 1 & 4 & 2 & 0 \\ 3 & 6 & -3 & -3 \\ 0 & -2 & 3 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 4 & 2 & 0 \\ 0 & 1 & \frac{3}{2} & \frac{1}{2} \\ 0 & 0 & 6 & 2 \end{pmatrix} \xleftarrow{2R_2 \rightarrow R_3} \begin{pmatrix} 1 & 4 & 2 & 0 \\ 0 & 1 & \frac{3}{2} & \frac{1}{2} \\ 0 & -2 & 3 & 1 \end{pmatrix} \xleftarrow{-\frac{1}{6}R_2} \begin{pmatrix} 1 & 4 & 2 & 0 \\ 0 & -6 & -9 & -3 \\ 0 & -2 & 3 & 1 \end{pmatrix}$$

$$\xrightarrow{\frac{1}{6}R_3} \begin{pmatrix} 1 & 4 & 2 & 0 \\ 0 & 1 & \frac{3}{2} & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{3} \end{pmatrix}$$

$$(a, b, c) = \left(-\frac{2}{3}, 0, \frac{1}{3}\right)$$

(104)