

Linear Algebra (matrix algebra)

We are getting a taste of L.A.

We won't prove much nor give underlying reasoning for why things are.

Linear algebra all about the study of systems of linear equations.

Linear eqn: $a_1x_1 + a_2x_2 + \dots + a_nx_n = b$

constants: $a_1, \dots, a_n, b$

variables: $x_1, \dots, x_n$

A system of L.E.s is a collection of them

$$\begin{cases} ax + by = c \\ dx + ey = f \end{cases}$$

Solve both simultaneously.

Real-world applications not uncommon to have millions of variables and also equations.

Ex: We expect constants a, b, c such that

$$y = ax^2 + bx + c$$

Goal: Find a, b , and c , where the parabola
(95) passes through $(1, 1)$, $(2, 4)$, $(-1, 1)$

$$\text{At } (1,1): \quad 1 = a + b + c \quad \left. \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right\}$$

$$\leftarrow 1 = 1 + 0 + c$$

$$c = 0$$

$$\text{At } (2,4): \quad 4 = 4a + 2b + c$$

$$\text{At } (-1,1): \quad 1 = a - b + c$$

~~Save the third point...~~

We already know ~~so~~ how to solve these sorts of systems.

$$(1) - (3)$$

$$\begin{array}{r} 1 = a + b + c \\ - (1 = a - b + c) \\ \hline \end{array}$$

$$0 = 2b \Rightarrow b = 0.$$

$$1 = a + c$$

(1)

$$4 = 4a + c$$

(2)

$$1 = a + c$$

(96)

$$(2) - (1)$$

$$\begin{array}{r} 4 = 4a + c \\ - (1 = a + c) \\ \hline \end{array}$$

$$3 = 3a \quad a = 1$$

We conclude the desired formula:

$$y = x^2$$

What we did is an ad hoc approach grounded in L.A.

Ex: Meal must contain: 7 units fat, 9 units of protein, 16 units of carbs.

Three foods:

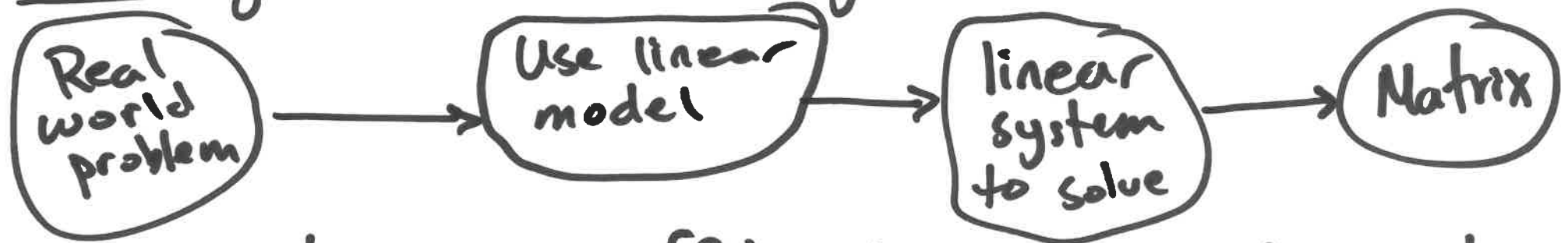
x	1.	2 fat,	2 protein,	4 carbs	/ 100g.
y	2.	3 fats,	1 protein,	2 carbs	" "
z	3.	1 fat,	3 protein,	5 carbs	" "

(97)

This can be realized as a solution to a linear ~~sys~~ system. What is that system?

$$\begin{cases} 2x + 3y + z = 7 & \text{(fat)} \\ 2x + y + 3z = 9 & \text{(protein)} \\ 4x + 2y + 5z = 16 & \text{(carb)} \end{cases}$$

Working with linear systems:



We build a (coeff.) matrix from the system.

$$A = \left(\begin{array}{ccc|c} 2 & 3 & 1 & 7 \\ 2 & 1 & 3 & 9 \\ 4 & 2 & 5 & 16 \end{array} \right)$$

Augmented
matrix b/c
RHS is in.

To solve the linear system, we apply operations onto the coeff. matrix — specifically row ops.
(Row ops b/c equations are rows.)

What are row operations? There are 3
fundamental row operations.

1. Multiply a row by a nonzero constant.
2. Interchange any two equations.

3. Add a constant multiple of one row to another.

All three do not change the solution to the linear system.

This is also called Gaussian Elimination.

