From this, we have that

$$242\cdot\underset{x}{(31)} + 1870\underset{y}{(-4)} = 22.$$

Thus, we have used Euclid's alg. to solve the Bezont equation.

Now suppose we want a solution to

$$242x + 1870y = 44.$$

We know $\gcd(242, 1870) = 22$ and $22 \mid 44$.
Therefore,

$$242(62) + 1870(-8) = 44.$$

How do we get all possible solutions to
$$ax+by=c\ ?$$

Thm. Let $a,b,c \in \mathbb{N}$, and set $d=\gcd(a,b)$. The equation
$$ax+by=c$$
has a solution if and only if $d \mid c$.
If (x_0,y_0) ~~are~~ is a solution, then every integer solution is of the form:
$$x = x_0 + \frac{b}{d}\cdot n\ , \qquad y = y_0 + \frac{a}{d}\cdot n$$
minus! $y=y_0-\frac{a}{d}n$

for all (and any) integer n.

Let's verify that all $x = x_0 + \frac{b}{d}n$, $y = y_0 - \frac{a}{d}n$ are solutions.

$$a\left(x_0 + \frac{b}{d}n\right) + b\left(y_0 - \frac{a}{d}n\right)$$

$$= ax_0 + \frac{ab}{d}n + by_0 - \frac{ab}{d}n$$

$$= ax_0 + by_0 = c. \qquad \text{(Because } (x_0, y_0) \text{ a sol.)}$$

With this new theory, try the apples and oranges problem again.

Procedure to get non-negative solutions.

$$ax + by = c.$$

1. Compute $d = \gcd(a, b)$.
2. Check $d \mid c$. (Assume $d \mid c$)
3. Use back-sub to ~~get one solution~~ solve the Bezout equation

$$ax + by = d$$

4. Multiply by $\frac{c}{d}$ to get a solution to $ax+by=c$
5. Write $X = X_0 + \frac{b}{d} n$, $y = y_0 - \frac{a}{d} n$
6. Impose $x \geq 0$, $y \geq 0$.

Modular Arithmetic

Def. Let $a, b, m \in \mathbb{Z}$, $m \geq 2$. We say that a is congruent to b modulo m, written as

notation!

$$a \equiv b \pmod{m},$$

if $a-b$ is an integer multiple of m.

In symbols,

there exists an integer k such that

$$a - b = k \cdot m.$$

The integer m is called the modulus.

Ex: $a = 9$, $b = ?$, $m = 5$.

If

$$a \equiv b \pmod{5},$$

then

$$a - b = 5k \quad \text{for some} \quad k \in \mathbb{Z}.$$

What values of b satisfy the congruence?

$b = 4, 9, 14, 19, 24, \ldots (+5), -1, \boxed{-6,} -11, -16, \ldots (-5)$

$$9 \equiv -6 \pmod{5}$$

$$9 - (-6) = 15 = 5 \cdot 3 \checkmark.$$