# MA 180 / 185 / 190

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Expectations:

- Group work participation.
- No tolerance disrespect.

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## ALGEBRA.

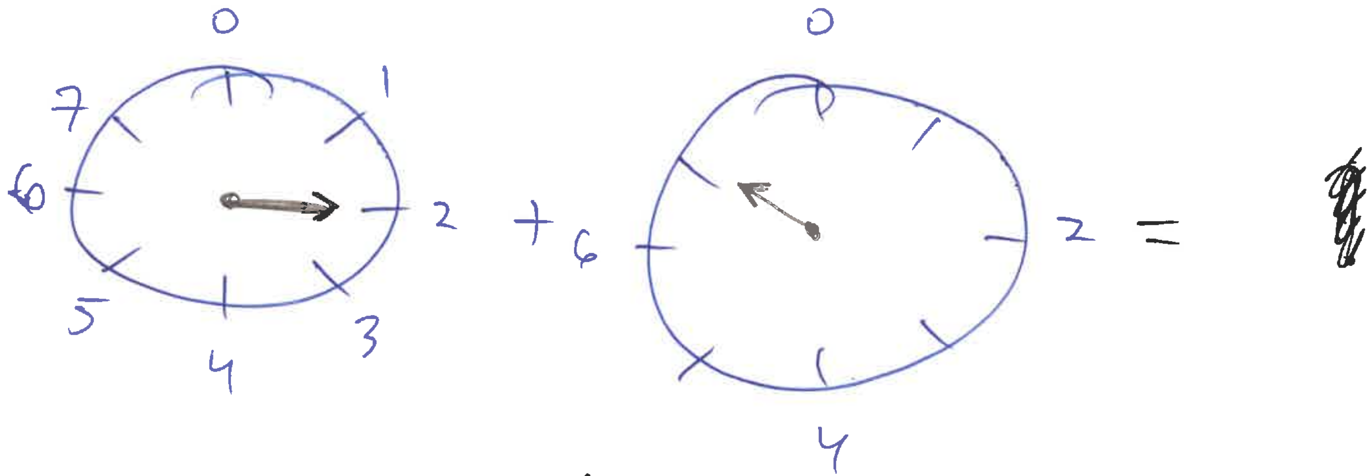
Study of systems of operations and relations.

Two topics in general

1. Modular arithmetic. %
2. Matrix algebra.

# Modular Arithmetic

"Clockwork arithmetic"




2 hours after 7:00 → 1:00.

## Applications

- Sanity checks: digits in UPCs, IBAN, CCNs, PPSN.
- Cryptography — Basically the way we keep passwords etc. safe over the internet.
- Error correction: sending information and fixing possible errors from transit.

# Classic & Simple Motivating problems.

**Notation.** A number $n$ satisfying

$$n \equiv 2 \pmod 3$$

"$n$ is equivalent to 2 modulo 3."

**Meaning:** Upon multiple divisions by 3 (of $n$) we have remainder 2.

Some $n$ that satisfy the equivalence:

$\boxed{n=2.}$ $\boxed{n=5.}$ $\frac{5}{3} = 1 + \frac{2}{3}$ remainder.

easy

$n = 8, 11, 14, \boxed{17,} \ldots, \quad 5, 2, -1, -4, -7, \ldots$

These are all the integers $n$ that satisfy the equivalence.

Suppose apples cost 69 cents and oranges cost 35 cents. The total is 2.78. If a is the # of apples and o the number of oranges, then

$$69a + 35o = 2.78 \qquad \text{(cents)}$$

Assuming a and o are integer amounts.

$\frac{69}{35} = 1 + \frac{34}{35}$

Reduce the equation modulo 35.

$$69a + 35o \equiv 278 \quad \boxed{(\text{mod } 35).}$$

$278 = 7 + \frac{33}{35}.$

Want to take remainders upon dividing by 35.

I.e. Replace the constants with their remainders.

$$34a \equiv 33 \quad (\text{mod } 35).$$

These two equations (mod 35) are "the same". They have the same solutions.

This is also the same as

$$(-1)\cdot a \equiv -2 \pmod{35}$$

$$-a \equiv -2 \pmod{35}.$$

$$a \equiv 2 \pmod{35}$$

Including this is important!

Multiply by $-1$

We conclude that a is 2 more than a multiple of 35. In symbols, there is an integer k such that

$$\boxed{a = 35\cdot k + 2.}$$

The possible number of apples is 2, 37, 72, .....

Because we spent only 278 cents on fruit, we know that $a=2$. $\Longrightarrow$ $o=4$.

# Numbers.

$\mathbb{N} = \{1, 2, 3, 4, \ldots\}$ natural numbers

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, 3, \ldots\}$ integers.

$\mathbb{Q} = \{ \frac{a}{b} \mid a, b \text{ are integers with } b \neq 0 \}$ rational numbers.

$\mathbb{R}$ : real numbers. $\sqrt{2}$ a non-rational but real.

$\mathbb{C}$ : complex numbers