Solutions: Algebra Test IID

1. a. True.

   b. False.

   c. True.

   d. True.

   e. False.

2. a. True.

   b. $x = 0$.

   c. $y = 5$.

   d. False.

   e. False.

3. a. The columns are $T(e_1)$, $T(e_2)$, $T(e_3)$:

$$\begin{pmatrix} 1 & 0 & 2 \\ 2 & 3 & -1 \end{pmatrix}.$$

   b. No. The matrix is 2×3, which is not square, so it is not invertible.

   c.

$$\begin{aligned} 2z &= -2, \\ 3y - z &= 3. \end{aligned}$$

   Therefore, $z = -1$ and $y = \frac{2}{3}$.

   d.

$$T\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 + 2 \cdot 1 \\ 2 \cdot 1 + 3 \cdot 2 - 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}.$$

4. a. No. The matrices have different sizes (2×2 and 2×3), so they cannot be added.

   b. Yes.

$$\begin{aligned} \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} &= \begin{pmatrix} 2 \cdot 1 + 3 \cdot 0 & 2 \cdot 3 + 3 \cdot 1 \\ 1 \cdot 1 + 2 \cdot 0 & 1 \cdot 3 + 2 \cdot 1 \end{pmatrix} \\ &= \begin{pmatrix} 2 & 9 \\ 1 & 5 \end{pmatrix}. \end{aligned}$$

   c. Yes.

$$\begin{aligned} \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix} &= \begin{pmatrix} 2 \cdot 1 + 3 \cdot 0 & 2 \cdot 1 + 3 \cdot 2 & 2 \cdot 2 + 3 \cdot 1 \\ 1 \cdot 1 + 2 \cdot 0 & 1 \cdot 1 + 2 \cdot 2 & 1 \cdot 2 + 2 \cdot 1 \end{pmatrix} \\ &= \begin{pmatrix} 2 & 8 & 7 \\ 1 & 5 & 4 \end{pmatrix}. \end{aligned}$$

d. Yes.

$$\begin{pmatrix} 1 & -1 \\ 4 & 2 \end{pmatrix} + \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 5 & 3 \end{pmatrix}.$$

5. a. False. Two of the equations give different values of t:

$$\begin{aligned} t + 1 - 2 = 1 \quad &\Longrightarrow \quad t = 2, \\ 3t - 2 \cdot 1 + 2 \cdot 2 = -7 \quad &\Longrightarrow \quad t = -3. \end{aligned}$$

b. True, with $t = 5$:

$$\begin{aligned} -1 + 5 - 3 &= 1, \\ 3 \cdot (-1) - 2 \cdot 5 + 2 \cdot 3 &= -7, \\ -2 \cdot (-1) + 3 \cdot 5 - 3 \cdot 3 &= 8. \end{aligned}$$

c. False. Two of the equations give different values of t:

$$\begin{aligned} 1 + t = 1 \quad &\Longrightarrow \quad t = 0, \\ 3 \cdot 1 - 2t = -7 \quad &\Longrightarrow \quad t = 5. \end{aligned}$$

d. True, with $t = -1$:

$$\begin{aligned} -1 + 1 - (-1) &= 1, \\ 3 \cdot (-1) - 2 \cdot 1 + 2 \cdot (-1) &= -7, \\ -2 \cdot (-1) + 3 \cdot 1 - 3 \cdot (-1) &= 8. \end{aligned}$$