MA180/MA185/MA190 Algebra Test II 19 November 2026

Name: _____

Student ID number: _____

Module code: ◯ MA180 ◯ MA185 ◯ MA190

(Optional:) LENS report: ◯ Yes ◯ No

Instructions. Provide your answers in the spaces provided. If you run out of space, extra paper will be provided. Be sure to include your name and student ID number on all additional sheets of paper that you submit.

If you have a LENS report that stipulates extra time, then please indicate this above and answer any four out of the five questions below.

All other students, please answer all five questions.

Question	**Score**
1	
2	
3	
4	
5	
Total	

1. ($10 = 2 + 2 + 2 + 2 + 2$ marks) Justification not required.

   (a) The following congruence holds: $13^{400} \equiv 1 \pmod{75}$

   ◯ True ◯ False

   (b) There are thirty-four integers in $\mathbb{Z}_{76}$ with a multiplicative inverse.

   ◯ True ◯ False

   (c) The following congruence holds: $2^{81} \equiv 2 \pmod{75}$

   ◯ True ◯ False

   (d) Suppose the function $f_E : \mathbb{Z}_{26} \to \mathbb{Z}_{26}$ defined by $f_E(x) = 11x + 3$ is used for encoding one-unit messages over a 26-letter alphabet. Then the decoding function is $f_D(x) = 19x + 21$.

   ◯ True ◯ False

   (e) Suppose the function $f_E : \mathbb{Z}_{26} \to \mathbb{Z}_{26}$ defined by $f_E(x) = 10x + 7$ is used for encoding one-unit messages over a 26-letter alphabet. Then the decoding function is $f_D(x) = 5x + 17$.

   ◯ True ◯ False

2. ($10 = 2+2+2+2+2$ marks) Justification not required.

(a) If $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation, then the image of the zero vector under T is always the zero vector.

◯ True ◯ False

(b) What must be true about x so that A is in reduced row echelon form

$$A = \begin{pmatrix} 1 & 4 & 0 & 0 \\ 0 & x & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}?$$

◯ $x = -4$ ◯ $x = -1$ ◯ $x = 0$ ◯ $x = 1$ ◯ $x = 4$

(c) Let $B = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 5 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & -5 & 1 \\ 0 & y & -1 \\ 0 & 1 & 0 \end{pmatrix}$. Find y such that $C = B^{-1}$.

◯ $y = -5$ ◯ $y = -1$ ◯ $y = 0$ ◯ $y = 1$ ◯ $y = 5$

(d) The following linear system admits a solution:

$$\begin{cases} x + 4y & = 2 \\ 3x + 12y & = 7 \end{cases}$$

◯ True ◯ False

(e) If $A = \begin{pmatrix} 4 & 3 \\ 2 & 2 \end{pmatrix}$, then $A^{-1} = \begin{pmatrix} 2 & -3 \\ -2 & 4 \end{pmatrix}$.

◯ True ◯ False

3. ($10 = 3 + 2 + 3 + 2$ marks) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the transformation given by:

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x + 2z \\ 2x + 3y - z \end{pmatrix}.$$

(a) Determine the matrix of the transformation T with respect to the standard basis of $\mathbb{R}^3$.

(b) Is the matrix found in part (a) invertible? Please explain. ◯ Yes ◯ No

(c) Find y and z such that $T\begin{pmatrix} 0 \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$.

(d) Determine the image under T of the point $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$.

4. ($10 = 2 + 3 + 3 + 2$ marks) For each of the following operations, determine whether it can be carried out or not. If it can be computed, do so. If it cannot, explain why.

(a) $\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix}$ ◯ Yes ◯ No

(b) $\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$ ◯ Yes ◯ No

(c) $\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix}$ ◯ Yes ◯ No

(d) $\begin{pmatrix} 1 & -1 \\ 4 & 2 \end{pmatrix} + \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$ ◯ Yes ◯ No

5. ($10 = 2 + 2 + 3 + 3$ (in some order) marks) Consider the following linear system

$$(*) \quad \begin{cases} x + y - z & = 1, \\ 3x - 2y + 2z & = -7, \\ -2x + 3y - 3z & = 8. \end{cases}$$

For each of (a)–(d) decide whether there exists t that completes the given vector to a solution of $(*)$ and compute such a t, or explain why it does not exist.

(a) $(t, 1, 2)$ can be completed to a solution of $(*)$ ◯ True ◯ False

(b) $(-1, t, 3)$ can be completed to a solution of $(*)$ ◯ True ◯ False

(c) $(1, t, 0)$ can be completed to a solution of $(*)$ ◯ True ◯ False

(d) $(t, 1, -1)$ can be completed to a solution of $(*)$ ◯ True ◯ False

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