Solutions: Algebra Test IIC

1. a.
$$T\begin{pmatrix}2\\-1\\1\end{pmatrix} = \begin{pmatrix}3\cdot(-1)+2\cdot 1\\3\cdot 2-(-1)+1\end{pmatrix} = \begin{pmatrix}-1\\8\end{pmatrix}.$$

   b.
$$\begin{aligned} 2z &= 4,\\ 3x+z &= 1. \end{aligned}$$
   Therefore, $z=2$ and $x=-\frac{1}{3}$.

   c. The columns are $T(e_1)$, $T(e_2)$, $T(e_3)$:
$$\begin{pmatrix}0 & 3 & 2\\3 & -1 & 1\end{pmatrix}.$$

   d. No. The matrix is 2×3, which is not square, so it is not invertible.

2. a. Yes.
$$\begin{pmatrix}1 & 3\\2 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 0\\2 & 1\end{pmatrix} = \begin{pmatrix}1\cdot 1+3\cdot 2 & 1\cdot 0+3\cdot 1\\2\cdot 1+1\cdot 2 & 2\cdot 0+1\cdot 1\end{pmatrix} = \begin{pmatrix}7 & 3\\4 & 1\end{pmatrix}.$$

   b. Yes.
$$\begin{pmatrix}1 & 3\\2 & 1\end{pmatrix}\cdot\begin{pmatrix}2 & 1 & 1\\1 & 0 & 3\end{pmatrix} = \begin{pmatrix}1\cdot 2+3\cdot 1 & 1\cdot 1+3\cdot 0 & 1\cdot 1+3\cdot 3\\2\cdot 2+1\cdot 1 & 2\cdot 1+1\cdot 0 & 2\cdot 1+1\cdot 3\end{pmatrix} = \begin{pmatrix}5 & 1 & 10\\5 & 2 & 5\end{pmatrix}.$$

   c. Yes.
$$\begin{pmatrix}3 & 1\\2 & -2\end{pmatrix}+\begin{pmatrix}1 & 4\\0 & 2\end{pmatrix} = \begin{pmatrix}4 & 5\\2 & 0\end{pmatrix}.$$

   d. No. The matrices have different sizes (2×2 and 2×3), so they cannot be added.

3. a. False.

   b. True.

   c. True.

   d. True.

   e. True.

4. a. False.

   b. True.

   c. $y=-7$.

d. $x = 0$.

e. False.

5. a. True, with $t = 2$:

$$\begin{aligned} -3 + 2 - 2 \cdot 2 &= -5, \\ 2 \cdot (-3) + 2 \cdot 2 + 3 \cdot 2 &= 4, \\ -(-3) - 2 + 4 \cdot 2 &= 9. \end{aligned}$$

b. False. Two of the equations give different values of t:

$$\begin{aligned} t + (-2) - 2 \cdot 4 = -5 \quad &\Longrightarrow \quad t = 5, \\ 2t + 2 \cdot (-2) + 3 \cdot 4 = 4 \quad &\Longrightarrow \quad t = -2. \end{aligned}$$

c. False. Two of the equations give different values of t:

$$\begin{aligned} 3 + 2 - 2t = -5 \quad &\Longrightarrow \quad t = 5, \\ 2 \cdot 3 + 2 \cdot 2 + 3t = 4 \quad &\Longrightarrow \quad t = -2. \end{aligned}$$

d. True, with $t = -3$:

$$\begin{aligned} 2 + (-3) - 2 \cdot 2 &= -5, \\ 2 \cdot 2 + 2 \cdot (-3) + 3 \cdot 2 &= 4, \\ -2 - (-3) + 4 \cdot 2 &= 9. \end{aligned}$$