Solutions: Algebra Test IIB

1. a. $x = 0$.

   b. False.

   c. False.

   d. $y = 5$.

   e. True.

2. a. True.

   b. True.

   c. False.

   d. True.

   e. True.

3. a. Yes.
$$\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix} + \begin{pmatrix} -2 & 1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}.$$

   b. Yes.
$$\begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 3 \cdot 2 + 1 \cdot 1 & 3 \cdot 0 + 1 \cdot 1 \\ 2 \cdot 2 + 2 \cdot 1 & 2 \cdot 0 + 2 \cdot 1 \end{pmatrix} = \begin{pmatrix} 7 & 1 \\ 6 & 2 \end{pmatrix}.$$

   c. No. The matrices have different sizes (2×2 and 2×3), so they cannot be added.

   d. Yes.
$$\begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 3 \cdot 2 + 1 \cdot 1 & 3 \cdot 1 + 1 \cdot 3 & 3 \cdot 0 + 1 \cdot 1 \\ 2 \cdot 2 + 2 \cdot 1 & 2 \cdot 1 + 2 \cdot 3 & 2 \cdot 0 + 2 \cdot 1 \end{pmatrix} = \begin{pmatrix} 7 & 6 & 1 \\ 6 & 8 & 2 \end{pmatrix}.$$

4. a. The columns are $T(e_1)$, $T(e_2)$, $T(e_3)$:
$$\begin{pmatrix} 2 & 0 & 1 \\ 1 & 3 & -1 \end{pmatrix}.$$

   b. No. The matrix is 2×3, which is not square, so it is not invertible.

   c.
$$T \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \cdot 1 + 2 \\ 1 + 3 \cdot (-1) - 2 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \end{pmatrix}.$$

   d.
$$\begin{aligned} 2x &= -2, \\ x + 3y &= 4. \end{aligned}$$

   Therefore, $x = -1$ and $y = \frac{5}{3}$.

5. a. False. Two of the equations give different values of t:

$$\begin{aligned} t + 2 \cdot 1 - 1 = -1 \quad &\Longrightarrow \quad t = -2, \\ -2t + 1 + 2 \cdot 1 = -3 \quad &\Longrightarrow \quad t = 3. \end{aligned}$$

b. True, with $t = -1$:

$$\begin{aligned} 4 + 2 \cdot (-1) - 3 &= -1, \\ -2 \cdot 4 + (-1) + 2 \cdot 3 &= -3, \\ 3 \cdot 4 - (-1) - 3 \cdot 3 &= 4. \end{aligned}$$

c. True, with $t = 3$:

$$\begin{aligned} 3 + 2 \cdot (-1) - 2 &= -1, \\ -2 \cdot 3 + (-1) + 2 \cdot 2 &= -3, \\ 3 \cdot 3 - (-1) - 3 \cdot 2 &= 4. \end{aligned}$$

d. False. Two of the equations give different values of t:

$$\begin{aligned} 4 + 2t - 1 = -1 \quad &\Longrightarrow \quad t = -2, \\ -2 \cdot 4 + t + 2 \cdot 1 = -3 \quad &\Longrightarrow \quad t = 3. \end{aligned}$$