## Solutions: Algebra Test IIA

1. a. Yes.

$$\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 2\cdot 1 + 1\cdot 2 & 2\cdot 0 + 1\cdot 1 & 2\cdot 2 + 1\cdot 3 \\ 1\cdot 1 + 3\cdot 2 & 1\cdot 0 + 3\cdot 1 & 1\cdot 2 + 3\cdot 3 \end{pmatrix}$$
$$= \begin{pmatrix} 4 & 1 & 7 \\ 7 & 3 & 11 \end{pmatrix}.$$

   b. No. The matrices have different sizes ($2 \times 2$ and $2 \times 3$), so they cannot be added.

   c. Yes.

$$\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 2\cdot 1 + 1\cdot 0 & 2\cdot 2 + 1\cdot 3 \\ 1\cdot 1 + 3\cdot 0 & 1\cdot 2 + 3\cdot 3 \end{pmatrix}$$
$$= \begin{pmatrix} 2 & 7 \\ 1 & 11 \end{pmatrix}.$$

   d. Yes.

$$\begin{pmatrix} 1 & 3 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 2 & -1 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 3 & 4 \end{pmatrix}.$$

2. a.

$$-2y = 4,$$
$$2y + 3z = 1.$$

   Therefore, $y = -2$ and $z = \frac{5}{3}$.

   b. The columns are $T(e_1)$, $T(e_2)$, $T(e_3)$:

$$\begin{pmatrix} 3 & -2 & 0 \\ 1 & 2 & 3 \end{pmatrix}.$$

   c. No. The matrix is $2 \times 3$, which is not square, so it is not invertible.

   d.

$$T\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 3\cdot 1 - 2\cdot 2 \\ 1 + 2\cdot 2 + 3\cdot(-1) \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}.$$

3. a. $y = -7$.

   b. False.

   c. False.

   d. True.

   e. $x = 0$.

4. a. True.

   b. True.

   c. False.

d. False.

e. True.

5. a. True, with $t = -2$:

$$\begin{aligned} -2 + 3 + (-2) &= -1, \\ 3 \cdot 2 + 2 \cdot 3 + 2 \cdot (-2) &= 8, \\ 2 \cdot 2 - 3 \cdot 3 - 3 \cdot (-2) &= 1. \end{aligned}$$

b. False. Two of the equations give different values of $t$:

$$\begin{aligned} -t + 3 + 1 = -1 \quad &\implies \quad t = 5, \\ 3t + 2 \cdot 3 + 2 \cdot 1 = 8 \quad &\implies \quad t = 0. \end{aligned}$$

c. True, with $t = 2$:

$$\begin{aligned} -2 + (-1) + 2 &= -1, \\ 3 \cdot 2 + 2 \cdot (-1) + 2 \cdot 2 &= 8, \\ 2 \cdot 2 - 3 \cdot (-1) - 3 \cdot 2 &= 1. \end{aligned}$$

d. False. Two of the equations give different values of $t$:

$$\begin{aligned} t + 1 = -1 \quad &\implies \quad t = -2, \\ 2t + 2 \cdot 1 = 8 \quad &\implies \quad t = 3. \end{aligned}$$