

Name: _____

Student ID number: _____

Module code: ☐ MA180 ☐ MA185 ☐ MA190(Optional:) LENS report: ☐ Yes ☐ No

Instructions. Provide your answers in the spaces provided. If you run out of space, extra paper will be provided. Be sure to include your name and student ID number on all additional sheets of paper that you submit.

If you have a LENS report that stipulates extra time, then please indicate this above and answer any four out of the five questions below.

All other students, please answer all five questions.

Question	Score
1	
2	
3	
4	
5	
Total	

1. ($10 = 2 + 2 + 2 + 2 + 2$ marks) Mark True or False. No justification needed.

(a) The following holds: $20 \mid 10$ ☐ True ☐ False

(b) The greatest common divisor of 24 and 60 is 6. ☐ True ☐ False

(c) There exist integers x, y such that $4x + 10y = 2^{100}$. ☐ True ☐ False

(d) There exists an integer x such that $3x \equiv 1 \pmod{48}$. ☐ True ☐ False

(e) If $d \mid (a \cdot b)$ for positive integers a, b, d , then $d \mid a$ or $d \mid b$. ☐ True ☐ False

2. ($10 = 2 + 2 + 3 + 3$ marks) For each of the following congruences, determine x as an integer in $\{0, 1, \dots, m - 1\}$ where m is the relevant modulus.

(a) $x \equiv 21 + 6 \pmod{8}$

(b) $x \equiv 86 \cdot 6 \pmod{9}$

(c) $13x \equiv -1 \pmod{10}$

(d) $5x \equiv 2 \pmod{9}$

3. (5 + 5 marks) (a) Use Euclid's algorithm to find $\gcd(48, 84)$.

- (b) Find an integer x in $\{0, 1, \dots, 83\}$ such that $48x \equiv 36 \pmod{84}$. Justify your answer.

4. (10 = 2 + 3 + 5 marks) (a) Decompose 132 as product of prime powers.

- (b) Compute $\phi(132)$, that is the number of integers between 0 and 131 that are coprime with 132.

- (c) Use Euler's Totient Theorem to find x in $\{0, 1, \dots, 131\}$ such that $x \equiv 7^{2442} \pmod{132}$.

5. ($10 = 2 + 2 + 2 + 4$ marks) Consider the following system of congruences

$$(*) \quad \begin{cases} x \equiv 3 \pmod{5} \\ x \equiv 5 \pmod{6} \\ x \equiv 2 \pmod{7} \end{cases} .$$

For each of (a), (b), and (c), indicate whether the given value of x satisfies the system of congruences. Briefly justify your answer. (Exactly one of the three is a solution.)

- (a) $x = 18$ is a solution to $(*)$ ☐ True ☐ False

- (b) $x = 23$ is a solution to $(*)$ ☐ True ☐ False

- (c) $x = 53$ is a solution to $(*)$ ☐ True ☐ False

- (d) Use the information above to find an integer between 200 and 350 that is a solution to the system. Briefly justify your answer.

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