PROBLEM SET 3

SUBMIT BY 29 NOVEMBER 2024

INSTRUCTIONS

The assignment should be submitted on Canvas by **5:00pm on 29 November 2024** as an ipynb file (i.e. a Jupyter notebook).

The ipynb should be machine readable, and it must reproduce your answers when I run it. Develop your own Python code rather than simply using existing Python modules for computing the relevant mathematics (e.g. statsmodels). If in doubt, send me an email. The ipynb file should contain your answers to the questions raised below in markdown (or "text") cells and not Python cells.

The use of generative AI is prohibited and will be filed, in line with the university's policy, as academic misconduct. Read more here:

https://www.universityofgalway.ie/academicintegrity/.

The homework will be graded according to a scheme in which *content* (i.e. correctness of your answers, choice of methods, python code) is weighted at 80% and *presentation* (i.e. manner in which you present your answers, methods, and code) is weighted at 20%.

Late submissions will be accepted at a 10% deduction every 24 hours past the deadline, starting with 10%. For example, submitting two hours after the deadline on Canvas will incur a 10% deduction, whereas 26 hours after the deadline will incur 20%.

PROBLEMS

Problem 1. Write your own Python function, called BasicKMeans, to compute a k-means clustering. The input and output of BasicKMeans should satisfy the following:

Input: a positive integer k, and a list (or other list-like iterable) of n data points in $\mathbb{R}^m$.

Output: a list L of length n such that L[i] = j implies the ith point is contained in the jth cluster.

Do not use any pre-built k-means algorithms.

Problem 2. The following table gives distances between vertices of a graph.

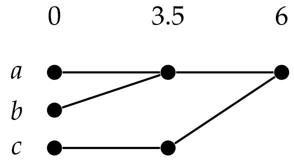
	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
v_1	0	9	6	1	6	7	8	2
v_2	9	0	3	8	8	4	6	10
v_3	6	3	0	6	6	2	4	8
v_4	1	8	6	0	5	7	7	2
v_5	6	8	6	5	0	7	3	6
v_6	7	4	2	7	7	0	5	9
v_7	8	6	4	7	3	5	0	8
v_8	2	10	8	2	6	9	8	0

Suppose $t \geqslant 0$, and let G_t be the graph with vertices $\{v_1, \ldots, v_8\}$ and edges $\{\{v_i, v_j\} \mid \mathrm{d}(v_i, v_j) \leqslant t\}$.

(1) Give the adjacency matrices for the three graphs: G_2, G_5, and G_8.

(2) Give two dendrograms that encodes the inclusion between the connected components of the graphs $G_0, G_1, \ldots, G_{10}$. One dendrogram should use single-linkage and the other dendrogram should use complete-linkage.

Note: Instead of drawing a dendrogram, we will represent the same information in Python as a list D of pairs (t, C), where t is a real number and C is a list of sets, each set in C corresponds to a cluster. For example the dendrogram



can be represented as

Date: November 20, 2024.

```
D = [
  (0, [{‘a’}, {‘b’}, {‘c’}]),
  (3.5, [{‘a’, ‘b’}, {‘c’}]),
  (6, [{‘a’, ‘b’, ‘c’}])
]
```