CS3101: ASSIGNMENT 4

DEADLINE: 29 MAR AT 17:00

Description: Create two different Jupyter notebooks (`ipynb` files). These should be self-contained, and all computations done exclusively in these notebooks.

Notebook 1: Write a SageMath function that takes three arguments: an integer n, an integer k, and a string t. The function should return the **polynomial expression** for the *wonderful* (n,k)*-polynomial* in the variable determined by t. The wonderful (n,k)-polynomial $W_{n,k}(t)$ is defined as follows:

$$W_{n,k}(t) = \begin{cases} \dfrac{(1-t^n)(1-t^{n-1})\cdots(1-t^{n-k+1})}{(1-t^k)(1-t^{k-1})\cdots(1-t)} & \text{if } 0 < k \leqslant n, \\ \dfrac{(1-t^{-2n})(1-t^{2-2n})\cdots(1-t^{2k-2n-2})}{(1-t^{-k})(1-t^{1-k})\cdots(1-t)} & \text{if } n \leqslant k < 0, \\ 1 & \text{if } k = 0, \\ 0 & \text{otherwise.} \end{cases}$$

Note: the first two expressions have exactly k factors in the numerator and denominator.

Use your function to do the following for each $m \in \{1, \ldots, 10\}$:

(i) determine the coefficient of t^{2m^2} in $W_{4m,2m}(t)$.

(ii) compute the difference $W_{2m,m}(t) - W_{-m,-m}(t)$.

Notebook 2: Write a function that takes an $n \times 3$ matrix with integer entries, for a positive integer n, and provides a report for the polytope P obtained from the convex hull of the rows of the given matrix. The function should **return the wireframe plot** of P. The report should print the following information.

(i) the list of vertices of P as vectors,

(ii) the volume of P, and

(iii) the centroid of P.

Use your function on two different 3-dimensional polytopes.

Submission: Submit only the two `ipynb` files. This can be done by uploading each file separately, or by putting the files into a `zip` file, which is then uploaded.

Grading: Some important points about the grading of this assignment.

- If the SageMath code raises errors, marks will be deducted.
- Marks will be deducted for omitting meaningful computations.

Date: March 1, 2024.