Unstable examples

Thursday, July 9, 2020 9:37 PM

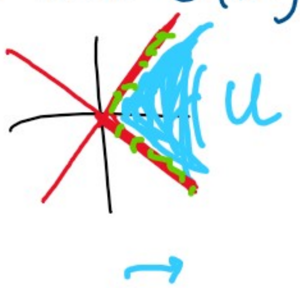
1. $x_0 \in \overline{\Omega'}$
2. $\forall x \in \Omega'$, $G(x) > 0$. $G_f'(x) > 0$
3. $G(x) = 0$ for all $x \in \partial\Omega' \cap \Omega$.

Ex 9.14.

$$x' = x + xy$$
$$y' = -2y + xy.$$

$(0,0)$.

Let $G(x,y) = x^2 - y^2$. Note that $G(x,y) = 0$ if and only if $|x| = |y|$.



Now we want to define suitable Ω and Ω'.

First

$$U = \{(x,y) \in \mathbb{R}^2 \mid x > |y|\}.$$

For all $(x,y) \in U$, $G(x,y) > 0$ and if $(x,y) \in \partial U$, then $G(x,y) = 0$.

$$G_f'(x,y) = (2x, -2y) \cdot (x + xy, -2y + xy)$$
$$= 2x^2 + 2x^2 y + 4y^2 - 2xy^2$$

$F(x,y) = 2x^2 y - 2xy^2$. We want to see how ~~small~~ negative this function gets.

The func. F has one critical point: $(0,0)$

The func. F has an ...

$F(0,0) = 0$.

We check F along boundary:

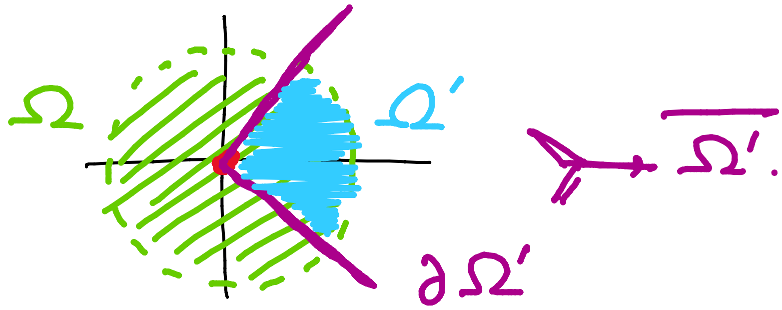
- $x = y$: $F(x,x) = 0$.
- $-x = y$: $F(x,-x) = -4y^3$. ← Bad.

We address this by bounding region.

$\Omega = B_1(0)$ and set

$$\Omega' = U \cap \Omega = \{(x,y) \in \mathbb{R}^2 : |y| < x < 1\}.$$

Therefore, the cond. of Prop 9.13 are met. Thus $(0,0)$ is an unstable fixed pt



Ex 9.15.

$$\left.\begin{aligned} x' &= x^2 + 2x^2 y, \\ y' &= xy + x^3. \end{aligned}\right\} (0,0).$$

$$G(x,y) = Ax^2 - By^2.$$

$$G_f'(x,y) = (2Ax, -2By) \cdot (x^2 + 2x^2 y,\ xy + x^3)$$

$$= 2Ax^3 + \underline{4Ax^3 y} - 2Bxy^2 - \underline{2Bx^3 y}$$

$$= 2Ax^3 + (4A - 2B)x^3 y - 2Bxy^2$$

$$= 2Ax^3 + \underbrace{(4A-2B)x^2y}_{0} - 2Bxy^2$$

Set $A=1$, $B=2$. $\quad G_f' = 2Ax^3 - 2Bxy^2$

$$= 2x\underbrace{(Ax^2 - By^2)}$$

$$\underbrace{G = x^2 - 2y^2}_{>0} \qquad G_f' = \underbrace{2x}_{>0}\underbrace{(x^2-2y^2)}_{>0}$$

Let $\Omega = B_1(0)$, and

$$\Omega' = \{(x,y) \in \Omega : x^2 > 2y^2,\ x>0\}.$$

This is open and satisfies all conds. of Prop 9.13. Thus x_0 is unstable. $\blacksquare$