Faculty for Mathematics
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Problem Set 6

Each problem is worth 2 points. The set is due on Wednesday 3 June by 23:59.

1. The two dynamical systems defined by the following functions have bifurcations that occur at $(a, x) = (0, 0)$

   (a) $f_a(x) = x + x^2 - a$,

   (b) $g_a(x) = x + x^2 - a^2$.

   Discuss how the stability of the system changes in a neighborhood around $a = 0$, and sketch the corresponding bifurcation diagrams.

2. Discuss how the stability of the system changes in a neighborhood around $a = 1$ for the system given by

$$f_a(x) = ax + x^4,$$

   and sketch the corresponding bifurcation diagram.

3. Using the Implicit Function Theorem in Theorem 4.4 (Saddle-node bifurcation), we proved the existence of a C^2-function $h : K \to \mathbb{R}$ such that

   (i) $h(\bar{x}) = \bar{a}$,

   (ii) for all $x \in K$, $F(h(x), x) = x$, and

   (iii) $h'(\bar{x}) = 0$.

   Show that

$$h''(\bar{x}) = -\frac{\frac{\partial^2 F}{\partial x^2}(\bar{a}, \bar{x})}{\frac{\partial F}{\partial a}(\bar{a}, \bar{x})}.$$

   Hint: Implicitly differentiate the expression $F(h(x), x) = x$ with respect to x. (Twice!)