Faculty for Mathematics
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Problem Set 2

Each problem is worth 2 points. The set is due on Wednesday 6 May by 23:59.

1. Define $f : \mathbb{N} \to \mathbb{N}$ by
$$f(x) = \begin{cases} 3x + 1 & \text{if } x \text{ is odd,} \\ x/2 & \text{if } x \text{ is even.} \end{cases}$$
   (a) Is f injective? Is f surjective? (Explain why.)
   (b) Describe the orbits of x for $x = 1, 2, 3, 4$ and 5.

2. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous and bijective function and consider the discrete dynamical system defined by $x_{n+1} = f(x_n)$.
   (a) Prove that any periodic point of f must have minimal period $p \leq 2$.
   (b) Prove that f has no eventually periodic points.

   Hint: First show that f is monotone. Consider the monotonically increasing case first. If f is monotonically decreasing, what can be said about f^2?

4. Determine the fixed point(s) of the dynamical systems determined by the family of functions
$$f_c(x) = x^2 + c.$$
   Hint: Consider the three cases $c < 1/4$, $c = 1/4$, and $c > 1/4$ separately.