Faculty for Mathematics
Bielefeld University

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Problem Set 12

Each problem is worth 2 points. The set is due on Wednesday 15 July by 23:59.

1. (a) Consider the system

    $$x_1' = 2x_1 - 8x_2$$
    $$x_2' = x_1 - 4x_2.$$

    Find all fixed points and discuss their stability. Write down the general solution to the system and sketch a phase diagram.

    (b) Now consider the system

    $$x_1' = 2x_1 + ax_2$$
    $$x_2' = x_1 - 4x_2.$$

    For which values of $a \in \mathbb{R}$ is the fixed point $x = 0$ asymptotically stable?

2. For each of the following systems, find the fixed points and discuss their stability. Then determine the stable and unstable subspaces if applicable.

    (a)

    $$x' = -3x + 4y,$$
    $$y' = -2x + 3y.$$

    (b)

    $$x' = 4x - 2y,$$
    $$y' = 5x + 2y.$$

3. Consider the system $x' = Ax$, where

    $$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}.$$

    (a) Find the general solution.
    (b) Find the fixed points and discuss their stability.
    (c) Sketch the phase diagram.

4. By finding a Lyapunov function for the system, show that $x = 0$ is a stable fixed point of

    $$x' = -2xy - 2y^2,$$
    $$y' = x^2 - y^3 + xy.$$

    Is $x = 0$ asymptotically stable?

    (*Hint*: Consider functions of the form $L(x, y) = A(x - x_0)^\alpha + B(y - y_0)^\beta$.)