

Problem Set 11

Each problem is worth 2 points. The set is due on Wednesday 8 July by 23:59.

1. Let $A = \begin{pmatrix} -1 & 2 \\ -4 & 5 \end{pmatrix}$.

(a) Find the polynomial q such that $e^A = q(A)$.

(b) Using part (a), determine e^A .

2. Find e^{tA} , where $t \in \mathbb{R}$, for

(a) $A = \begin{pmatrix} 2 & -3 \\ 3 & -4 \end{pmatrix}$ and

(b) $A = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$.

3. Let

$$A = \begin{pmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix}.$$

(a) Show that $A^n = (-1)^{n+1}A$ for all $n \in \mathbb{N}$.

(b) Use part (a) to calculate e^{tA} .

4. Let $A = \begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix}$. Determine the flow for the system $x' = Ax$ passing through $(4, -1)$.