Lecture 1

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Chapter 1: Introduction

Fundamental idea:
Under the future based on current state of conditions.

Def: A system is a set of measurable quantities, and a dynamical system is a system changing in time.

We use models to analyze.

Def: The state of a system is a set of values describing the system at time.

- The state space: all poss. states
- The time evolution rule: the function that describes how the system changes over time.

Two kinds of systems.

Def: A discrete time dynamical system is a dynamical system with discrete time steps: example $t_0, t_1, t_2, \dots$

Def: A continuous time dynamical system is a dyn. sys. that changes continuously with time.

Continuous DS: differential eq.
Discrete DS: difference eq.

1.1: Newton-Raphson Method

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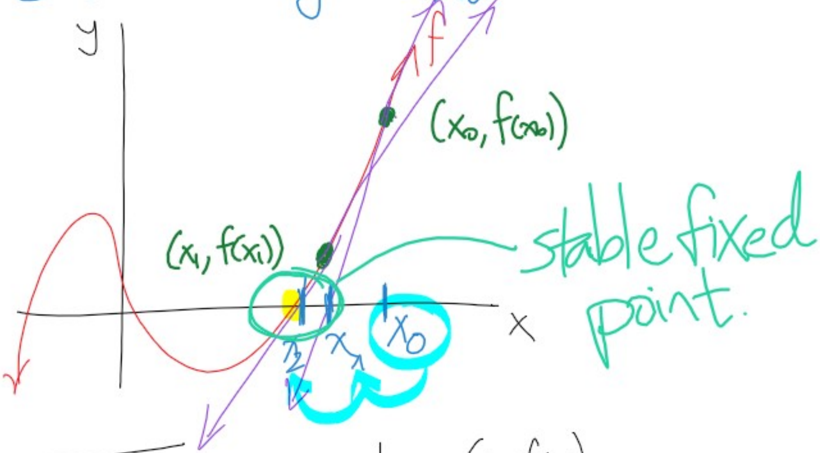
Method to find zeros/roots of functions $f: \mathbb{R} \to \mathbb{R}$.

Suppose f is diff. function, and we want a root ($\bar{x} \in \mathbb{R}$, $f(\bar{x}) = 0$) near some initial guess x_0.



Start with x_n, we have $(x_n, f(x_n))$

Want tangent line of f thru P.

Call this $T_n(x) = mx + c$.

$$m = f'(x_n) \qquad f(x_n) = m x_n + c$$

$$c = f(x_n) - f'(x_n) x_n$$

$$T_n(x) = f'(x_n)(x - x_n) + f(x_n).$$

The intersection is $0 = T_n(x)$.

$$x_{n+1} := x = x_n - \frac{f(x_n)}{f'(x_n)}$$

This yields a discrete DS.

initial guess is x_0

the next is $x_1, x_2, \ldots$

1.2: Exponential Growth / Decay.

Discrete. $a \in \mathbb{R}^+$ $(a > 0)$, $x_0 \in \mathbb{R}$, for all $n \in \mathbb{N}$,

$$\boxed{x_{n+1} = a \cdot x_n} \xrightarrow[\text{equation}]{\text{difference}} \overset{\text{diff. eq.}}{\boxed{x_{n+1} - x_n = a x_n - x_n}}$$

$$\boxed{x_{n+1} = a \cdot x_n} \xrightarrow[\text{equation}]{\text{difference}} \boxed{\begin{aligned} x_{n+1} - x_n &= a x_n - x_n \\ &= (a-1)x_n \\ &= \lambda x_n \end{aligned}}$$

$x_1 = a \cdot x_0$

$x_2 = a x_1 = a(a x_0) = a^2 x_0$

$x_3 = a \cdot x_2 = a^3 x_0$

$$\boxed{x_n = a^n x_0}$$

Want to understand long term behavior.

Case 1: $a = 1$. $x_n = x_0$

Case 2: $a > 1$.

- $x_0 = 0$. System always 0. This is a fixed point.
- $x_0 > 0$. $\boxed{x_{n+1} > x_n}$
  $x_0 < x_1 < x_2 < \cdots$
  $(x_n)_{n=0}^{\infty}$ is monotonically increasing. $\to \infty$
- $x_0 < 0$. $x_{n+1} < x_n < 0$
  $x_0 > x_1 > x_2 > \cdots$
  $(x_n)_{n=0}^{\infty}$ is mon. dec. $\to -\infty$

Going away from $x=0$

$x=0$ is unstable fixed point

Case 3: $0 < a < 1$.

- $x_0 = 0$. Then system is fixed at $x=0$, so $x=0$ is a f.p.
- $x_0 > 0$. $\boxed{x_{n+1} < x_n}$
  $x_0 > x_1 > x_2 > \cdots$ $(x_n) \to 0$
  mon. dec. all pos. $x_n > 0$
- $x_0 < 0$. $x_{n+1} > x_n$.
  $x_0 < x_1 < x_2 < \cdots$ $(x_n) \to 0$
  mon. inc. all neg. $x_n < 0$

Going towards $x=0$.

$x=0$ is a stable fixed point

Cont. 1. x ||

Cont. $\frac{dx}{dt} = \overset{\dot{x}}{\overset{\parallel}{x'}} = \lambda x$

x depends only on t, and this D.E describes how x changes.